2783. Proposed by C. C. BRAMBLE, U. S. Naval Academy.

Two players A and B take turns throwing a single die, A leading. The one first making a score of 3 aces is to be the winner. Find the probability that A will win.

416 (Algebra) [May, 1914]. Proposed by C. E. FLANAGAN, Wheeling, Va.

The sides of a given rectangle are a and b in which a rectangle is to be inscribed one of whose sides is c. Find the other side, using Euler's rule for quartics.

353 (Calculus) [February, 1914]. Proposed by R. P. LOCHNER, Philadelphia, Pa.

The center of a sphere, radius $R = 5$ inches, is $a = 10$ inches above the surface of a sphere, radius $12\frac{1}{2}$ inches. There is a point of light at $b = 1$ inch horizontally from a point $c = 10$ inches vertically above the surface of the first sphere. What is the area of the shadow which the upper sphere casts on the lower one?

287 (Mechanics) [February, 1914]. Proposed by W. H. DRANE, Lebanon, Tenn.

While sitting in an empaled enclosure, I noticed that the spokes of the wheels of passing automobiles, when viewed through the pickets of the fence, appeared to revolve more slowly than they really did, and in some instances even appeared to be revolving in a direction opposite to that in which they were really turning. Explain this optical illusion.

202 (Number Theory) [December, 1913]. Proposed by A. R. SCHWEITZER, Chicago, Ill.

There exists an infinitude of systems of dyads $\{\alpha\beta\}$ in 7, 9, 11, etc., elements such that each system has the following properties: (1) if $\alpha\beta$ is in the set, then $\beta\alpha$ is not in the set; (2) for each dyad $\alpha\beta$ in the set there exists an element ξ such that $\xi\beta$ and $\alpha\xi$ are also in the set. For example, such a system is,

12,	23,	34,	45,	56,	67,	78,	89,	91
13,	24,	35,	46,	57,	68,	79,	81,	92
14,	25,	36,	47,	58,	69,	71,	82,	93
51,	62,	73,	84,	95,	16,	27,	38,	49.

Investigate the existence of

I. A finite set of triads $\{\alpha\beta\gamma\}$ such that (1) if $\alpha\beta\gamma$ is in the set, then $\beta\gamma\alpha$, $\gamma\alpha\beta$ are also in the set but $\beta\alpha\gamma$ is not in the set; (2) for each triad $\alpha\beta\gamma$ in the set there exists an element ξ such that $\xi\beta\gamma$, $\alpha\xi\gamma$, $\alpha\beta\xi$ are also in the set.

II. A finite set of tetrads $\{\alpha\beta\gamma\delta\}$ such that (1) if $\alpha\beta\gamma\delta$ is in the set, then $\beta\gamma\alpha\delta$, $\gamma\alpha\beta\delta$, $\gamma\delta\alpha\beta$ are also in the set but $\beta\alpha\gamma\delta$ is not in the set; (2) for each tetrad $\alpha\beta\gamma\delta$ in the set there exists an element ξ such that $\xi\beta\gamma\delta$, $\alpha\xi\gamma\delta$, $\alpha\beta\xi\delta$, $\alpha\beta\gamma\xi$ are also in the set.

The problem for alternating n-ads for $n > 4$ is obvious.

SOLUTIONS OF PROBLEMS.

2699, 2710 [May, June, 1918; April, 1919]. Proposed by the late R. E. MOORE, University of Wisconsin.

If $a_k^{(r)}$ denotes the kth term of an arithmetic progression of order r, and c_k denotes the kth binomial coefficient in the expansion of $(a - b)^n$ (n being a positive integer), show that

$$s \equiv \sum_{k=1}^{n+1} c_k a_k^{(r)} = 0, \text{ if } n > r.$$

II. SOLUTION BY A. PELLETIER, Montreal, Canada.

The terms of the arithmetic progression may be expressed in the usual form as follows:

$$a, \qquad a + \Delta_1, \qquad a + 2\Delta_1 + \Delta_2, \qquad a + 3\Delta_1 + 3\Delta_2 + \Delta_3, \qquad \cdots.$$

We have to prove that

$$c_0 a + c_1(a + \Delta_1) + c_2(a + 2\Delta_1 + \Delta_2) + c_3(a + 3\Delta_1 + 3\Delta_2 + \Delta_3) + \cdots + c_n\left(a + n\Delta_1 + \frac{n(n-1)}{1\cdot 2}\Delta_2 + \cdots\right) = 0,$$